

(Forthcoming in *Australasian Journal of Philosophy*)

Parallels Between Gaps and Gluts

Daniel Immerman

Abstract: This paper compares two attempted solutions to the liar paradox, both of which involve revisions to classical semantics. The first, that of truth value gaps, denies that all sentences are true or false. The second, that of truth value gluts, asserts that some sentences are true and false. A natural question is: what is the relationship between these two solutions; are they both equally good (or bad) solutions, or is one better than the other? In 1990, Terence Parsons offered an answer to this question when he published an article titled ‘True Contradictions’. In it, Parsons argued that for every problem for one solution, there is a parallel problem for the other. Since then, several people have given arguments for and against the idea that the two solutions face parallel problems. This paper attempts to advance the debate in two ways. First, it answers four attempts by Graham Priest to reject the parallel. In particular, it discusses alleged advantages for the gap solution with regards to the teleological account of truth, an extended liar paradox, Berry’s paradox, and inexpressibility. Second, in the conclusion it discusses where, if anywhere, the parallel may be broken.

Keywords: *Truth Value Gaps, Truth Value Gluts, Liar Paradox*

1. Introduction

In 1990, Terence Parsons wrote an article called ‘True Contradictions’ in which he argued that two solutions to the liar paradox, that of truth value gaps and that of truth value gluts, face parallel problems. Since then, several people have given arguments for and against the idea that these two solutions face parallel problems.¹ This paper answers four attempts by Graham Priest to reject the parallel. Then, in the conclusion, it discusses where, if anywhere, the parallel may be broken.

But first, some background on the two solutions and the alleged parallel between them. Here is the liar sentence:

⟨L⟩: ⟨L⟩ is false.

This generates the liar paradox: what is the truth value of ⟨L⟩? The glut solution asserts that ⟨L⟩ is true and false; that is, it admits of truth-value gluts. In order to motivate and explain the gap solution discussed in this paper, we shall first consider a different gap solution, which asserts that ⟨L⟩ is neither true nor false. This solution is faulty because it cannot handle the following revenge liar paradox:

¹ See in particular [Priest 1995] and [Restall 2004].

⟨R⟩: ⟨R⟩ is not true.

Suppose our gapper asserts that ⟨R⟩ is neither true nor false. Then, according to ⟨R⟩ itself, ⟨R⟩ is true. So that ⟨R⟩ is neither true nor false cannot be maintained. One way for the gapper to fix her problem is to take advantage of a distinction between acceptance and rejection, and a parallel distinction between assertion and denial. Graham Priest writes:

Acceptance and rejection (as I shall use the terms – of course they can be used in other ways) are cognitive states. To accept something is simply to believe it, to have it in one’s “belief box”, as it were. To reject something is to refuse to believe it ... Accepting something and rejecting it would certainly seem to be exclusive; but they are not exhaustive. One might not believe something because one has never considered it, or because one has considered it but found no evidence, or found insufficient evidence. ... Assertion and denial, unlike acceptance and rejection, are not cognitive states, but kinds of linguistic acts. ... Assertion and denial are closely connected with cognitive states, however. Crudely, they are linguistic expression of acceptance and rejection, respectively [Priest 2006a: 104].

As they shall be used in the rest of this paper, ‘acceptance’ and ‘rejection’ are exclusive but not exhaustive, and likewise for ‘assertion’ and ‘denial.’ Some have rejected this, but discussion of this point is beyond the scope of this paper.

A better gap solution does not assert that ⟨R⟩ is neither true nor false. Rather, it rejects that ⟨R⟩ is true and it rejects that ⟨R⟩ is false; that is to say, it rejects that ⟨R⟩ is true or ⟨R⟩ is false. Such a solution avoids the revenge liar discussed above. This is the solution that will be discussed in the rest of this paper.

The glutter, the gapper, and someone who accepts classical semantics will all say different things about various languages. This paper is written in English. Things will be said that are arguably inexpressible, or at least very difficult to express, in the language of glutters and gappers.

The glutter can be thought of as denying the law of non-contradiction. He asserts that some sentences are true and false. The gapper can be thought of as denying the law of excluded middle. She denies that all sentences are true or false. This does not mean that she introduces some third truth value. There are only two truth values, true and false. It just happens to be the case, on the gapper’s semantics, that some sentences do not have either.

In his article, Parsons noted an interesting parallel between gappy and glutty solutions. If you take statements regarding one solution and replace ‘true and false’ with ‘true or false,’ ‘assert’ with ‘deny,’ ‘accept’ with ‘reject,’ ‘for all’ with ‘there exists’ and so on you get the other.² Here are some examples of parallel statements: 1) The glutter accepts that the liar sentence is true and false. The gapper rejects that the liar sentence is true or false. 2) The glutter asserts that all sentences are true or false. The gapper denies that there exist sentences that are true and false.

² What is the extent of the parallel; how many pairs like this are there? It is unclear; this paper adds two more: ‘least’ and ‘greatest’ and ‘rational’ and ‘irrational.’

Using this parallel, Parsons argued that for every problem for the gapper, there is a parallel problem for the glutter, and vice versa. In various places, Graham Priest has denied this. The next four sections will examine four problems that Priest thinks apply only to the gapper and respond to them in favor of the parallel.

2. Priest's Teleological Account of Truth

The first place in which Priest denies the parallel is connected with a certain account of truth that Priest thinks is correct, which he calls the 'teleological account'. He thinks that the gap solution cannot use the account to ground a definition of 'false', while the glut solution can. To be more precise, Priest accepts that, for both the gapper and the glutter, 'false' can be defined in terms of truth and negation. In particular, a sentence is false iff its negation is true. The problem is that 'not' is just as in need of definition as 'false' is. So Priest's position, more precisely, is that the teleological account of truth cannot be used to ground a definition of falsity for the gapper unless negation has already been defined.

The story is complicated. In the first edition of *In Contradiction*, Priest offered the teleological account of truth: 'Asserting, like other human activities, has a telos or point, and the telos of asserting is truth' [Priest 2006b: 61]. Then, he used the teleological account of truth to argue that all sentences are either true or false:

... if a sentence is not true, it is false ... This fact about falsity follows from the analysis of truth we have just had. To speak truly is to succeed in a certain activity. And in the context of asserting, anything less than success is failure. There is no question of falling into some limbo between the two [Priest 2006b: 64].

Parsons, in 'True Contradictions,' criticizes this argument:

The main difficulty with this argument is that it fails to establish the point. If the goal of asserting is truth, and if we fail, we have asserted something that is not a truth. That is, we have asserted some sentence $\langle \alpha \rangle$ where $\langle \alpha \rangle$ isn't true. That's failure. But the point that Priest needs to establish is not that $\langle \alpha \rangle$ isn't true, but rather that the negation of $\langle \alpha \rangle$ is true [Parsons 1990: 342].

In the second edition of *In Contradiction*, Priest admits that his original argument had this flaw: 'I think that Parson's [*sic*] critique of the argument is right' [Priest 2006b: 267]. Nonetheless, in both his article, 'Gaps and Gluts: Reply to Parsons', and in the second edition of *In Contradiction*, Priest suggests that Parsons has missed the force of the argument. In the former, he writes:

In a multiple-person game brought to a legitimate conclusion, if a person doesn't win, she fails; but we can distinguish within the category of failures according to whether some other person wins (loss) or whether no other person does (draw). In

a single-person game (such as patience) brought to a legitimate conclusion, however, there is no intrinsic ground for distinguishing between two different sorts of outcome. So it is for asserting, for assertion is a single-person game ... We may still try to distinguish, within the category of failures, two subcategories. However, as the single-game analogy shows, there is no intrinsic ground for doing this; hence any such distinction would be spurious, and so could not ground any important semantic distinction [Priest 1995: 59].

Priest's argument here is something like the following. We are to find definitions of truth and falsehood by considering assertion. Assertion is a type of single-person game. In a single-person game, there are only two options, success or failure. Truth is associated with success at the game of assertion, falsity with failure. If, like the gapper, we do not associate falsity with all failures, then there is no way for us to define falsity in terms of assertion.

In the second edition of *In Contradiction* Priest adds the following:

The [original argument that Parsons criticized] shows that, so far as the linguistic act of assertion goes, there is nothing to distinguish between different ways in which an utterance may fail to meet its end. ... It therefore (at the very least) raises a challenge for the gappist to find some other ground [Priest 2006b: 267].

This, too, suggests that Priest's original argument is mostly a challenge to the gapper to find a basis in assertion for falsehood.

My response: this is as much of a problem for the glitter as for the gapper. Suppose the glitter tries to define falsity as the failure of an assertion. The glitter says that some sentences are both true and false. This means, according to the single-person game analogy, that these sentences are both successes and failures at the same time. According to the way we normally think of single-person games, this hardly makes any sense. Thus, it appears that neither the gapper nor the glitter can define falsity as a failed assertion.³

3. The Extended Liar and Assertion

Priest's next argument is that there is an extended liar paradox, due to G. Littman, that is more problematic for the gapper than the glitter. Priest does not say whether the argument is supposed to be valid in classical, gappy, or glutty logic, but probably it is supposed to be valid at least in classical logic. Here it is:

Consider the claim β : it is irrational to assert $\langle\beta\rangle$. Suppose that someone asserted $\langle\beta\rangle$. They would then be asserting something, and at the same time asserting that it is irrational to assert it. This is irrational. Hence, asserting $\langle\beta\rangle$ is irrational. But

³ The gapper, it will be noted, has to reject that success and failure is exhaustive, while the glitter must accept that it is not exclusive. But Priest has given no reason to think that the former is more problematic than the latter.

this is just $\langle\beta\rangle$, and we have established it. Hence, it is rational to assert $\langle\beta\rangle$ [Priest 1995: 61].⁴

As stated, β is not fully clear. In particular, it is ambiguous between ‘it is always irrational to assert $\langle\beta\rangle$ ’, ‘it is sometimes irrational to assert $\langle\beta\rangle$ ’, and ‘this assertion of $\langle\beta\rangle$ is irrational’. On the first two readings, the argument is invalid.⁵ So, we must choose the third option:

$\langle\alpha\rangle$: this assertion of $\langle\alpha\rangle$ is irrational.

Let us examine Priest’s argument more carefully. We start by numbering the steps:

1. Suppose that someone asserts $\langle\alpha\rangle$.
2. They would then be asserting something, and at the same time asserting that it is irrational to assert it.
3. It is irrational to assert something and at the same time assert that it is irrational to assert it.
-
4. Asserting $\langle\alpha\rangle$ is irrational.
5. But this is just $\langle\alpha\rangle$, and we have established it.
-
6. It is rational to assert $\langle\alpha\rangle$.

This argument seems to be missing a premise between 5 and 6:

5.5 if $\langle\alpha\rangle$ is true and we have established it, then it is rational to assert it.

Now, so far, the conclusion of this argument seems to be:

7. It is rational to assert $\langle\alpha\rangle$ and it is irrational to assert $\langle\alpha\rangle$.

So, as it stands, the argument’s conclusion is not a dialethia, but rather a surprising statement about rationality. In order to make it into a dialethia, we need the following premise:

⁴ An anonymous referee thinks this argument is ambiguous with regards to whether assertion must be rationally coherent and that either disambiguation is problematic. If so, then both gappers and glutters should reject this argument for this reason, and thus it favors neither over the other.

⁵ In particular, let $\langle\gamma\rangle$ be it is always irrational to assert $\langle\gamma\rangle$. Suppose that someone asserted $\langle\gamma\rangle$. Then they would be asserting it, and at the same time asserting that it is always irrational to assert it. This is irrational. But that has not established $\langle\gamma\rangle$ because it might, at other times, be rational to assert $\langle\gamma\rangle$.

Let $\langle\delta\rangle$ be it is sometimes irrational to assert $\langle\delta\rangle$. Suppose that someone asserted $\langle\delta\rangle$. They would then be asserting something, and at the same time, asserting that it is sometimes irrational to assert it. It need not be irrational to utter such a sentence – perhaps this time is not one of those times that it is irrational to assert it, so Priest’s argument doesn’t go through.

8. It is rational to assert $\langle \alpha \rangle$ if and only if it is not the case that it is irrational to assert $\langle \alpha \rangle$.

This yields the conclusion:

9. It is both true and false that it is rational to assert $\langle \alpha \rangle$.

Priest thinks the argument favors the glutter over the gapper. Probably, his reason is something like the following. The argument is valid in classical logic. Thus, to respond to it, the gapper and glutter must either reject a premise or accept the conclusion.⁶ The glutter can accept the conclusion, which is a violation of the law of non-contradiction, and use his resources to explain why he is justified in accepting it. However, there is no premise that is an instance of the law of excluded middle. Thus, in rejecting a given premise, the gapper will not be able to use her resources to explain why she is justified in rejecting it.

In response, I will offer a parallel argument, such that, however the first argument favors the glutter over the gapper, the parallel favors the gapper over the glutter. In particular, the argument is classically valid. But it contains a premise which is an instance of the law of the excluded middle, so the gapper can reject this premise, and use her resources to explain why she is justified in rejecting it. However, the argument's conclusion is not a violation of the law of non-contradiction, and thus the glutter cannot use his resources to explain why he is justified in accepting it.

First, let $\langle \zeta \rangle$ be as follows:

$\langle \zeta \rangle$: This denial of $\langle \zeta \rangle$ is rational.

It is important to note that one can deny $\langle \zeta \rangle$ in the course of uttering it. This might seem surprising because $\langle \zeta \rangle$ looks more like an assertion than a denial. However, note the following. Denial is a type of speech act. One way of signaling that one is engaging in the speech act of denial is to utter the words 'I deny' before a sentence. For example: someone in a court room might utter the words 'I deny that I am guilty'. This person may be said to have denied that she is guilty. Likewise, one can deny $\langle \zeta \rangle$ by uttering the words 'I deny that this denial of $\langle \zeta \rangle$ is rational'. Here is the parallel argument:

⁶ It is important to note that neither the gapper nor glutter can offer the following diagnosis: the argument is invalid, all of its premises are true and not false, and its conclusion is false and not true. This might sound somewhat surprising. After all, some arguments that are valid in classical logic are invalid in gappy and glutty logics. So, one might think that the gapper or glutter can simply argue that the argument is invalid according to his or her preferred logic and be done.

Here is why this idea will not work. If an argument is classically valid, then, according to the semantics of gappers and glutters, so long as none of its sentences have truth value gaps or gluts, it is truth-preserving. This is because the gappers and glutters have the same rules for the connectives as the classical logician does when it comes to sentences with the values 'true' and 'false.' The way in which they differ from classical logician is that they have added rules to handle sentences that have both truth values or lack either truth value. So, it is not enough for a gapper or glutter to respond to such an argument by noting that it is invalid. So long as none of the sentences have truth value gaps or gluts, the argument will still be truth preserving. Thus, in order to 'solve' the argument the gapper or glutter has to either reject one of the premises or accept the conclusion.

- A. Suppose someone denies $\langle \zeta \rangle$.
- B. It is either true or false that this denial of $\langle \zeta \rangle$ is rational.
- C. Suppose it is false that this denial of $\langle \zeta \rangle$ is rational.
- D. Then $\langle \zeta \rangle$ is false.
- E. It is rational to deny false sentences.
- F. Then it is rational to deny $\langle \zeta \rangle$.

—

G. It is true that $\langle \zeta \rangle$ is rational to deny. (By B and the Argument from C to F)

—

H. It is sometimes rational to deny something and at the same time deny that it is rational to deny it. (By G)

This conclusion, H, is implausible. Imagine someone denying something and at the same time denying that it was rational to deny it. Then they would be denying that their very denial was rational. H says that what this person is doing is rational, but this is implausible.

H is the dual of one of the premises in Priest's original argument, namely premise 3: 'It is irrational to assert something and at the same time assert that it is irrational to assert it'. H is just as implausible, for the classical logician, as premise 3 is plausible.

This argument contains a premise which is an instance of the law of excluded middle (namely B) and its conclusion is not an instance of a violation of the law of non-contradiction. Thus, it favors the gapper in the same way Priest's original argument favored the glutter. The parallel has been maintained.

4. Berry's Paradox

Another argument Priest has given in favor of glut solutions is that they can handle certain semantic paradoxes that gap ones cannot, for instance, Berry's paradox. Priest's statement of Berry's paradox favors the glutter over the gapper in exactly the same way Priest's other argument did. In particular, its conclusion is an instance of a violation of the law of non-contradiction, but it does not contain any premises which are instances of the law of excluded middle. My response is the same: to offer a parallel argument which contains an instance of the law of excluded middle as a premise, but whose conclusion is not an instance of the law of non-contradiction. But first, Berry's paradox:

English has a finite vocabulary. Hence there is a finite number of noun phrases with less than 100 letters. Consequently there can be only a finite number of natural numbers which are denoted by a noun phrase of this kind. Since there is an infinite number of natural numbers, there must be numbers which are not so denoted. Hence there must be a least. Consider the least number not denoted by a noun phrase with fewer than 100 letters. By definition, this cannot be denoted by

a noun phrase with fewer than 100 letters, but we have just so denoted it.
Contradiction [Priest 2006b: 16].

As before, the reason why this argument is problematic for the gapper, according to Priest, is that it does not contain, as a premise, an instance of the law of excluded middle. In particular, Priest has given a couple of formal proofs for Berry's paradox, ending in a contradiction, that do not use the law of excluded middle as one of their premises [Priest 1995: 160-5; Priest 2006b: 25].⁷

The gapper can respond by giving the following dual argument:

1. It is either true or false that: for every number denoted by a noun phrase with fewer than 100 letters, there is a greater number denoted by a noun phrase with fewer than 100 letters.
2. Suppose it is true.
3. Then there are infinitely many numbers denoted by noun phrases with fewer than 100 letters.
4. Then there are infinitely many noun phrases with fewer than 100 letters.
5. Suppose it is false.
6. Then there is a number denoted by a noun phrase with fewer than 100 letters such that there is no greater number denoted by a noun phrase with fewer than 100 letters.
7. We have just denoted it.
8. Nonetheless, by 5, we cannot denote the number one greater than it using a noun phrase with fewer than 100 letters.
-
9. Premise 4 or Premise 8 is true.

This argument contains, as a premise, an instance of the law of excluded middle (Premise 1), but its conclusion is not a dialethia, but merely an implausible statement. In particular, 4 is clearly implausible; after all, there are a finite number of letters in English. And 8 is also implausible. If 6 denotes a number, then surely the following does as well: 'the number one greater than the number denoted in 6'.

More to the point, 4 and 8 are each as implausible as various premises in Berry's paradox are plausible. 4 is just as implausible as the following premise is plausible: there are finitely many noun phrases with fewer than 100 letters. And likewise, 8 is just as implausible as the following premise is plausible: 'the least number that cannot be denoted by a noun phrase greater than 100 letters' denotes a number.

Again, the parallel has been maintained.

5. Dialetheism and Expressibility

⁷ R.T. Brady argued that Priest's first proof relied on the law of excluded middle, so Priest offered the second proof in response [Priest 1996b: 25]. If Berry's paradox relied on the law of excluded middle, Priest could not use it to gain a dialectical advantage over the gapper. For, just as he could accept the conclusion, she could reject a premise.

Another problem Priest has raised concerns the gapper's use of denials. Priest thinks the gapper needs to embed denials within sentences in order to state her view. He gives, as an example, the following sentence:

⟨***⟩: If ⟨ α ⟩ is not true, any conjunction ⟨ $\alpha \wedge \beta$ ⟩ is not true [Priest 1995: 64].

Presumably, such a sentence is being used to explain the truth-tables for conjunction. In particular, the gapper here is explaining what the truth value of ⟨ $\alpha \wedge \beta$ ⟩ is in the case in which ⟨ α ⟩ is not true. Priest explains why such a sentence is problematic. 'The negations in this cannot be understood as denials since they are not attached to whole utterances (which force operators must be); and understood as negations, the claim is untrue' [Priest 1995: 64]. That is, there are certain things that we can express in English, which it seems like it would be nice for the gapper to express, such as ⟨***⟩. But the only way the gapper can express that a sentence lacks a truth value is to deny it and its negation. Thus, without being able to embed denials in sentences, the gapper will not be able to express ⟨***⟩.⁸

There are parallel problems for the glutter. Suppose Priest wishes to express:

⟨!⟩: If ⟨ α ⟩ is true, then ⟨ $\neg\alpha$ ⟩ is false.

Just to be clear, this sentence expresses the idea that if ⟨ α ⟩ is simply true, that is, true and not false, then ⟨ $\neg\alpha$ ⟩ is simply false, that is, false and not true. (It should be noted that here, we are not speaking in glutterese – the glutter cannot express the idea of simply true – or more precisely, if the glutter defines simply true in his own language, it will turn out that simply true sentences can still be false. This is a parallel, for ⟨* * *⟩ as Priest himself notes, cannot be expressed by a gapper. The problem we are considering here is whether the gapper and glutter can describe their semantics from within their own language. In both cases, we have picked a piece of their semantics, and are trying to see if they can express it adequately within their language.)

If a glutter like Priest utters ⟨!⟩, he has not fully expressed his view. For there is no way for glutters to express notions of simply true and simply false. Suppose there were. Let 'T' stand for simply true. Then, consider the revenge liar ⟨R⟩.

⟨R⟩: ⟨R⟩ is not T.

If ⟨R⟩ is T then ⟨R⟩ is false. If ⟨R⟩ is false then ⟨R⟩ is T. So, either way, ⟨R⟩ is T and false. So 'T' does not stand for simply true.

Thus, the only way that Priest can express that a sentence ⟨ ϕ ⟩ is not true is to reject ⟨ ϕ is true⟩. We can use the reject operator REJECT to express what Priest needs to say, in order to describe this piece of gapper semantics.

⁸ The reason using negations will not work is the following: if α and β are both the liar sentence, then according to the gapper semantics "if α is false, then ⟨ $\alpha \wedge \beta$ ⟩ is false" lacks a truth value. Thus, according to the gapper semantics, ⟨***⟩, read with negations, lacks a truth value.

$\langle ! \rangle$ If $\langle \alpha \rangle$ is true and REJECT[$\langle \alpha \rangle$ is false], then $\langle \alpha \rangle$ is false and REJECT[$\langle \alpha \rangle$ is true].

Thus the glutter faces a parallel problem here, for he, too, needs to embed rejections within larger sentences.

There are two ways in which Priest might try to escape this problem. Firstly, take the example:

SB: Snow is black.

Priest thinks there is nothing one can assert (without denying anything) that rules out a commitment to SB. For, suppose there were something that one could assert (without denying anything) to rule out a commitment to SB, let us call this G. Now, imagine someone who is willing to assert everything, while denying nothing. Such a person is willing to assert G, but in their case, asserting this (without denying anything) does not rule out a commitment to SB. So one cannot use asserting G (without denying anything) to rule out a commitment to SB. Thus, there is nothing one can assert (without denying anything) to rule out a commitment to SB [Priest 1995: 62].

What one can do, according to Priest, is assert a sentence that ensures that commitment to SB will lead to a ‘collapse into triviality’ [Priest 1995: 62]. In particular, one can assert $SB \rightarrow F$, where \rightarrow is a detachable conditional and F is ‘everything is true’ [Priest 1995: 62].⁹ Priest’s idea is that one’s assertion of $SB \rightarrow F$ will not rule out a commitment to SB. However it will ensure that if one is committed to SB, then one is committed to everything being true.

Priest has not tried to use this to solve the puzzle having to do with $\langle ! \rangle$, but it seems like it could be an option for him. However, as we shall see, it would not work. All Priest has told us about this conditional, so far, is that it is detachable. This means that, so long as the glutter wants to avoid a collapse to triviality, the only $\langle \alpha \rangle$ for which he can truly assert $(\neg \langle \alpha \rangle) \rightarrow F$ are $\langle \alpha \rangle$ that are simply true. But the question is, can he assert such sentences for all $\langle \alpha \rangle$ that are simply true?

Here is a dilemma. Suppose he cannot truly assert this of all $\langle \alpha \rangle$ that are simply true. Then, he cannot use this new idea to express $\langle ! \rangle$, because $\langle ! \rangle$ is supposed to range over all $\langle \alpha \rangle$ that are simply true.

Suppose he can assert this of all $\langle \alpha \rangle$ that are simply true. Then, he will be faced with a problem, which I will explain in English. First, define $\langle \mu \rangle$ as follows:

$\langle \mu \rangle$: $\langle \mu \rangle \rightarrow F$

According to the glutter’s semantics, $\langle \mu \rangle$ is either true, false, or both true and false. Suppose $\langle \mu \rangle$ is true or both. Then, in particular, $\langle \mu \rangle$ is true. Then $\langle \mu \rangle \rightarrow F$ is true. Then F is true. Suppose that $\langle \mu \rangle$ is simply false. Then, according to the glutter’s semantics, $\neg \langle \mu \rangle$ is simply true. It follows that

⁹ A conditional, ‘ \rightarrow ,’ is detachable if and only if for all x and y $(x \wedge (x \rightarrow y)) \vdash y$.

$\langle \mu \rangle \rightarrow F$ is true, which means that $\langle \mu \rangle$ is true. Because \rightarrow is a detachable conditional, this means that F is true. Thus, whatever the truth value that the glutter's semantics gives $\langle \mu \rangle$, F is true. This is bad, because the glutter wants to avoid F being true.

Thus, if the glutter introduces a symbol \rightarrow such that he will accept $(\neg\langle \alpha \rangle) \rightarrow F$ for all α that are simply true, then on his semantics, F will be true as well. This is not something he wants. In summary, either using this device cannot help the glutter assert $\langle ! \rangle$ or F is true on the glutter's semantics. In short, asserting sentences like ' $\langle \alpha \rangle \rightarrow F$ ' will not help the glutter avoid his failure to be able to express $\langle ! \rangle$.

Another option for Priest is to say that 'simply false' and 'simply true' are not clear notions. Priest adopts this strategy while criticizing a paper by Stewart Shapiro:

Shapiro goes on to worry about the notions of 'simple truth and simple falsehood', that is, by definition, being true but not false, or false but not true ($T\langle \alpha \rangle \wedge \neg F\langle \alpha \rangle$ and $F\langle \alpha \rangle \wedge \neg T\langle \alpha \rangle$). He says (p. 341) that to express the notions in those very words will 'not do the required work'. Exactly what the required work is supposed to be is not stated, and is unclear to me [Priest 2006b: 292].

In other words, Shapiro has noted that he cannot express his notions of 'simply true' and 'simply false' in glutterese. When he tries to define them in glutterese, he gets something different from what he wished. Priest responds that it is unclear which notions Shapiro wished to express. According to Priest, while Shapiro thinks he has a clear notion in mind when he talks about 'simple truth,' in fact, he does not.

The gapper can adopt a parallel response to Priest's original challenge. Recall that the gapper was challenged to embed ' $\langle \alpha \rangle$ is not true' within a conditional, where 'not true' meant something like fails to be true. And recall that she could not do this because she could not express the notion 'not true' semantically. But the gapper can now copy Priest. The gapper first notes that when Priest tries to define his notion in gapperese as not true ($\neg T$) what he gets is not what he wanted. Priest had meant to define the notion of failing to be true, which included both sentences that are neither true nor false and also sentences that are false, but what he has defined instead is merely falsehood. The gapper can then say that it is unclear what Priest meant to express by 'not true.' Thus, the parallel between the gapper and the glutter has been maintained. Both can say that the reason they cannot express their respective conditional is because the notions included in the conditional are obscure.¹⁰

In summary, the glutter and the gapper face parallel problems with regards to embedding rejections in conditionals, and this is because both face parallel problems expressing certain notions. And both can take parallel responses to their problems, namely to say the notions they have been asked to express don't make sense.

6. Conclusion

¹⁰ Indeed, not only are the gapper and the glutter here in a parallel position, the fundamental problem seems to be that neither can express the notion (if such a notion exists) of 'failing to be true.' For if the glutter could express this notion, then she could define 'simply false' as false and failing to be true.

Priest has failed to find problems for the gapper for which there are not parallel problems for the glut. What is the immediate significance of this? Firstly, it means that however serious the problems were that Priest raised for the gapper, there are equally serious problems for the glut. So, it appears that prospects look worse for the glut than they did before.

How far does this parallel go? Does every problem or advantage for one theory correspond to an equally weighty problem or advantage for the other theory? There are two reasons to think that the answer is no. Firstly, sections 3 and 4 relied on parallels between 'irrational' and 'rational' and between 'greater than' and 'less than'. These pairs of words are opposites, which suggests that if one wishes to find a problem or advantage for one theory that does not have a parallel, one should find a problem or advantage, the statement of which uses words that lack opposites. For example, some people argue that sentences like 'The present king of France is bald' lack a truth value, because they presuppose something false. It is unclear what the parallel of this would be, because it is unclear what the opposite of presupposing is.

Furthermore, even if each problem or advantage for one theory corresponds to a problem or advantage for the other theory, this does not show that the problems are of equal weight. For example, 1) the glut solution accepts that there are sentences which are true and false, whereas 2) the gap solution denies that all sentences are true or false. Despite the parallel, I personally find 2) a lot easier to stomach. Thus, another option would be to find problems or advantages for one solution that carry more weight than their parallels do for the other solution.¹¹

University of Notre Dame

References

- Parsons, Terence 1990. True Contradictions, *Canadian Journal of Philosophy* 20/3: 335-354.
- Priest, Graham 1983. Logical Paradoxes and the Law of Excluded Middle, *Philosophical Quarterly* 33/131, 160-5.
- Priest, Graham 1995. Gaps and Gluts: Reply to Parsons, *Canadian Journal of Philosophy* 25/1: 57-66.
- Priest, Graham 2006a. *Doubt Truth to Be a Liar*, Oxford: Clarendon Press.
- Priest, Graham 2006b. *In Contradiction*, 2nd. edition. Oxford: Clarendon Press.
- Restall, Greg 2004. Assertion, Denial, Accepting, Rejecting, Symmetry, Paradox, and All That. Given at the annual meeting of the Australasian Association of Philosophy. <<<http://consequently.org/writing/assertiondenialparadox>>>.

¹¹ Thanks for help to Paddy Blanchette, Graham Priest, Greg Restall, two anonymous referees, and participants in a logic seminar at Notre Dame.